MIMS DC
2023

4794/3

FACULTY OF SCIENCE

B.A./B.Sc. (I Semester) Examination

MATHEMATICS

Paper I

(Differential and Integral Calculus)

Time : 3 *Hours*] [*Max. Marks* : 100

Section A – (Marks : 8 × 5 = 40)

(Short Answer Questions)

1. Answer any **eight** questions :

(a) If $u = \log(\tan x + \tan y + \tan z)$, prove that

$$(\sin 2x)\frac{\partial u}{\partial x} + (\sin 2y)\frac{\partial u}{\partial y} + (\sin 2z)\frac{\partial u}{\partial z} = 2.$$

(b) If $u = f(r)$ where $r = \sqrt{x^2 + y^2}$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f^{11}(r) + \frac{1}{r} f^1(r)$.

(c) If $z = \sec^{-1}\frac{x^3 + y^3}{x + y}$, show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2\cot z$.

(d) If $x^y = y^x$ then find $\frac{dy}{dx}$.

(e) Expand the function $f(x, y) = x^2 + xy + y^2$ by Taylor's series in powers of $(x - 2)$ and $(y - 3)$.

(f) Determine the minimum value of $x^2 + y^2 + z^2$ subject to the condition $x + 2y - 4z = 5$.

(g) Find the radius of curvature of $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at the point where the line $y = x$ cuts it.

(h) Find the envelope of the family of curves $y = mx + am^3$ where m is the parameter.

(i) Show that $\frac{ds}{d\theta} = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}$ for the polar equation $r = f(\theta)$.

(j) Find the length of the arc of the parabola $y^2 = 4ax$ cut-off by its Latus rectum.

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(k) Find the volume of the solid obtained by revolving one arc of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$ about X-axis.

(l) Find the surface of a sphere of radius a.

Section B – (Marks : 4 × 15 = 60)

(*Essay Type Questions*)

Answer **all** *questions.*

2. (a) If $u = \sin^{-1}\left\{\dfrac{x^{\frac{1}{3}} + y^{\frac{1}{3}}}{x^{\frac{1}{3}} + y^{\frac{1}{3}}}\right\}^{\frac{1}{2}}$, show that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{144}(13 + \tan^2 u).$$

Or

(b) State and prove Euler's theorem and its Corollary on Homogeneous functions.

3. (a) If $x\sqrt{(1-y^2)} + y\sqrt{(1-x^2)} = a$ then show that $\dfrac{d^2y}{dx^2} = \dfrac{a}{(1-x^2)^{3/2}}$.

Or

(b) Prove that of all rectangular parallelopiped of the same volume, the cube has the least surface.

4. (a) If C_x and C_y be the chords of curvature parallel to the coordinate axes at any point of the curve $y = ae^{x/a}$, prove that $\dfrac{1}{C_x^2} + \dfrac{1}{C_y^2} = \dfrac{1}{2aC_x}$.

Or

(b) Show that the whole length of the evolute of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ is $4\left(a^2/b - b^2/a\right)$.

5. (a) Show that the volume of the solid obtained by revolving about X-axis the area enclosed by the parabola $y^2 = 4ax$ and its evolute $27ay^2 = 4(x-2a)^3$ is $80\pi a^3$.

Or

(b) State and prove Pappus theorem of Volume of Revolution.